Generalized Parton Distributions and the structure of the constituent quark [1]

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In a scenario where the constituent quarks are composite systems, Generalized Parton Distributions (GPDs) are built from wave functions to be evaluated in a Constituent Quark Model (CQM), convoluted with the GPDs of the constituent quarks themselves. The approach permits to access the full kinematical range corresponding to the DGLAP and ERBL regions, so that cross sections for deeply virtual Compton scattering can be estimated.

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1 Introduction

Generalized Parton Distributions (GPDs) [1] parametrize the non-perturbative hadron structure in hard exclusive processes [2]. The measurement of GPDs would represent a unique way to access crucial features of hadron structure [3, 4]. Relevant experimental efforts to measure GPDs, by exclusive electron Deep Inelastic Scattering (DIS), will take place soon and it is urgent to produce predictions for these quantities. Recently, calculations have been performed in Constituent Quark Models (CQM) [5, 6]. The CQM has a long story of successful predictions in low-energy studies of the structure of the nucleon. At high energies, in order to compare predictions with data, one has to evolve, according to perturbative QCD, the leading twist component of the structure functions obtained at the low-momentum scale, the "hadronic scale" μ_0^2, associated with the model. Such a procedure, already addressed in [7], has proven successful in describing the gross features of standard Parton Distribution Functions (PDFs)

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(see, e.g., [8]). Similar expectations motivated the study of GPDs in Ref. [5], where a simple formalism has been proposed to calculate the quark contribution to GPDs. The procedure of Ref. [5] has been extended and generalized in Ref. [9]. As a matter of fact, the approach of Ref. [5], when applied in the forward case, has been proven to be able to reproduce the gross features of PDFs [8] but, in order to achieve a better agreement with data, it has to be improved. In a series of papers, it has been shown that DIS data are consistent with a low energy scenario dominated by composite constituent quarks of the nucleon [10], defined trhough a scheme suggested by Altarelli, Cabibbo, Maiani and Petronzio (ACMP) [11], properly updated. The same idea has been recently applied to show the evidence of complex objects inside the proton [12], analyzing data of electron scattering off the proton. In this talk, we review the main findings of Ref. [9], where the same idea has been applied to obtain realistic predictions for GPDs and, at the same time, to explore kinematical regions, not accessible with the approach of Ref. [5]. In particular, the evaluation of the sea quark contribution becomes possible, so that GPDs can be calculated in their full range of definition. Such an achievement will permit to estimate the relevant cross-sections, providing us with a tool for planning future experiments.

2 GPDs in a constituent quark scenario

We are interested in hard exclusive processes, such as Deeply Virtual Compton Scattering (DVCS). We use the formalism of Ref. [4]. Let us think now to a nucleon target, with initial and final momenta P and P', respectively, with $\Delta = P' - P$ being the momentum transfer. The main quantity we discuss is the GPD $H(x, \xi, \Delta^2)$. In a system of coordinates where the virtual photon 4-momentum, $q^\mu = (q_0, \vec{q})$, and $\bar{P} = (P + P')^\mu/2$ are collinear along z, the ξ variable, the so called "skewedness", parameterizing the asymmetry of the process, is $\xi = -n \cdot \Delta$, where n is a light-like 4-vector with $n \cdot \bar{P} = 1$. With this choice, GPDs describe the amplitude for finding a quark with momentum fraction $x + \xi/2$ (in the Infinite Momentum Frame) in a nucleon with momentum $(1 + \xi/2)\bar{P}$ and replacing it back into the nucleon with a momentum transfer Δ^μ. Besides, when the quark longitudinal momentum fraction x of the average nucleon momentum $\bar{P}$ is less than $-\xi/2$ (the so-called negative DGLAP region), GPDs describe antiquarks; when it is larger than $\xi/2$ (the positive DGLAP region), they describe quarks; when it is between $-\xi/2$ and $\xi/2$ (the ERBL region), they describe $q\bar{q}$ pairs. One should keep in mind that, besides the variables x, ξ and Δ^2, GPDs depend, as the standard PDFs, on the momentum scale Q^2 at which they are measured. For an easy presentation, such a dependence will be omitted. In [5], the Impulse Approximation (IA) expression for $H_q(x, \xi, \Delta^2)$, suitable to perform CQM calculations, has been obtained. In particular, it has been shown that, taking into account the quarks degrees of freedom only, considering a process with $\vec{\Delta}^2 \ll M^2$, in a non relativistic

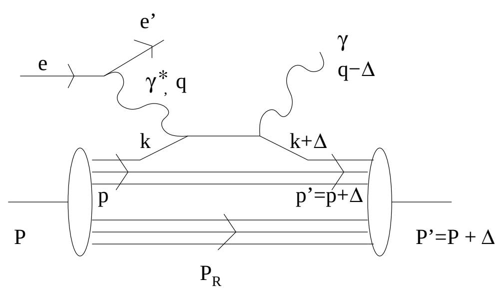

Figure 1: The DVCS process in the present approach.

framework one has

$$
\begin{aligned}
H_q(x,\xi,\Delta^2) &= \int d\vec{k}\ \delta\left(x+\frac{\xi}{2}-\frac{k^+}{\bar{P}^+}\right)\tilde{n}_q(\vec{k},\vec{k}+\vec{\Delta}) \\
&= \int d\vec{k}\ \delta\left(x+\frac{\xi}{2}-\frac{k^+}{\bar{P}^+}\right)\int e^{i((\vec{k}+\vec{\Delta})\vec{r}-\vec{k}\vec{r}')}\rho_q(\vec{r},\vec{r}')d\vec{r}d\vec{r}'\ , \quad (1)
\end{aligned}
$$

where the one-body non diagonal charge density $\rho(\vec{r},\vec{r}')$ and the one-body non-diagonal momentum distribution $\tilde{n}_q(\vec{k},\vec{k}+\vec{\Delta})$ have been introduced. The above equation allows the calculation of $H_q(x,\xi,\Delta^2)$ in any CQM, and it naturally verifies the two crucial constraints of the GPDs [5]. With respect to Eq. (1), a few caveats are necessary.

i) If use is made of a CQM, containing only constituent quarks only the DGLAP region can be explored. In order to introduce the study of the ERBL region, so that observables can be calculated, the model has to be enriched.

ii) in Eq. (1), the x variable for the valence quarks is not defined in its natural support. Several prescriptions have been proposed in the past to overcome such a difficulty in the standard PDFs case [7, 8].

iii) The Constituent Quarks are assumed to be point-like.

The two issues i) and ii) will be now discussed, by relaxing the condition iii) and allowing for a composite structure of the constituent quark

The procedure described in the previous section, when applied in the standard forward case, has been proven to be able to reproduce the gross features of PDFs [8]. In order to achieve a better agreement with data, the approach has to be improved. In a series of previous papers, it has been shown that DIS data are consistent with a low-energy simple picture of the constituent quark as a complex system of point-like partons. [10], updating the ACMP scenario [11].

Following the same idea, we describe here a model for the reaction mechanism of an off-forward process, such as DVCS. As a result, a convolution formula giving the proton H_q GPD in terms of a constituent quark off-forward momentum distribution, H_{q_0}, and of a GPD of the constituent quark q_0 itself, $H_{q_0 q}$, will be derived. It is assumed that the hard scattering with the virtual photon takes place on a parton of a spin 1/2 target, made of complex constituents. The

scenario we are thinking to is depicted in Fig. 1 for the special case of DVCS. In addition to the kinematical variables, x and ξ, already defined, one needs few more ones to describe the process. In particular, x' and ξ', for the "internal" target, i.e., the constituent quark, have to be introduced [9]. Using IA, a standard procedure can applied and a convolution formula, is readily obtained (see Ref. [9] for details):

$$H_q(x,\xi,\Delta^2) \simeq \sum_{q_0} \int_x^1 \frac{dz}{z} H_{q_0}(z,\xi,\Delta^2) H_{q_0 q}\left(\frac{x}{z},\frac{\xi}{z},\Delta^2\right) \tag{2}$$

where $H_{q_0}(z,\xi,\Delta^2)$ is to be evaluated in a given CQM, according to Eq. (1), for $q_0 = u_0$ or d_0, while $H_{q_0 q}(\frac{x}{z},\frac{\xi}{z},\Delta^2)$ is the constituent quark GPD, which is still to be modelled. We can start modelling this quantity thinking first of all to its forward limit, where "constituent quark parton distributions" have to be recovered. As we said, in a series of papers [10] a simple picture of the constituent quark as a complex system of point-like partons has been proposed, re-taking the ACMP scenario, [11].

Let us recall the main features of that idea. The constituent quarks are complex objects whose structure functions are described by a set of functions $\phi_{q_0 q}(x)$ that specify the number of point-like partons of type q which are present in the constituent of type q_0, with fraction x of its total momentum. We will call these functions the structure functions of the constituent quark. The functions describing the nucleon parton distributions are expressed in terms of the independent $\phi_{q_0 q}(x)$ and of the constituent density distributions ($q_0 = u_0, d_0$) as,

$$q(x,Q^2) = \sum_{q_0} \int_x^1 \frac{dz}{z} q_0(z,Q^2) \phi_{q_0 q}\left(\frac{x}{z},Q^2\right) \ , \tag{3}$$

where q labels the various partons, i.e., valence quarks (u_v, d_v), sea quarks (u_s, d_s, s), sea antiquarks ($\bar{u}, \bar{d},\ \bar{s}$) and gluons g. The different types and functional forms of the structure functions of the constituent quarks are derived from three very natural assumptions [11]: i) The point-like partons are QCD degrees o freedom, i.e. quarks, antiquarks and gluons; ii) Regge behavior for $x \to 0$ and duality ideas; iii) invariance under charge conjugation and isospin. The last assumption of the approach relates to the choice of the scale at which the constituent quark structure is defined. We choose $\mu_0^2 = 0.34$ GeV^2 [8, 14]. This hypothesis fixes *all* but one the parameters of the approach. The remaining paramter is fixed according to the value of F_2 at low x[11], and its value is chosen again according to [14]. The values of the parameters obtained are listed in [10]. The other ingredients appearing in Eq. (3), i.e., the density distributions for each constituent quark, are defined according to Eq. (1). Now we have to generalize this scenario to describe off-forward phenomena. Of course, the forward limit of our GPDs formula, Eq. (2), has to be given by Eq. (3): this gives $H_{q_0 q}(\frac{x}{z},\frac{\xi}{z},\Delta^2)$ [9].

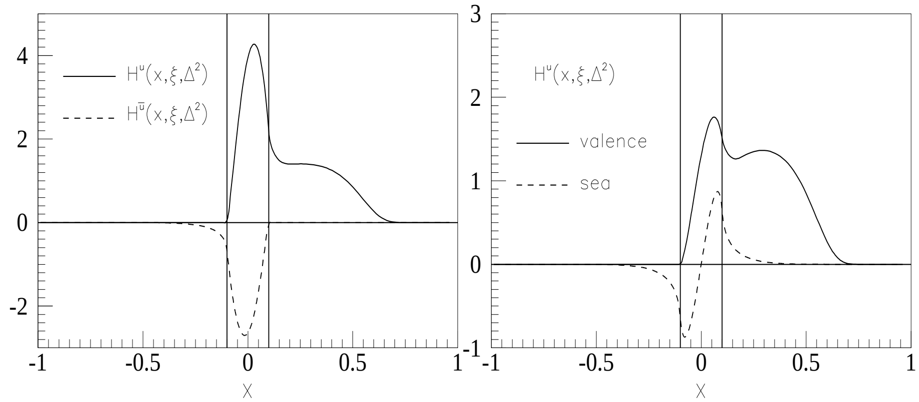

Figure 2: Left: the u-quark (full) and u-antiquark (dashed) H. Right: the u valence (full) and sea (sea-quarks plus antiquarks)(dashed) H GPD. Results are shown for $\xi = 0.2$ and $\Delta^2 = -0.5$ GeV^2, at the scale of the model.

Now the off-forward behaviour of the Constituent Quark GPDs has to be modelled. This can be done in a natural way by using the "α-Double Distributions" (DD's) language proposed by Radyushkin [13]. DD's, $\Phi(\tilde{x}, \alpha, \Delta^2)$, represent an alternative parametrization of the matrix elements describing DVCS and hard exclusive electroproduction processes, with respect to the one based on GPDs. The DD's do not depend on the skewedness parameter ξ; rather, they describe how the *total*, P, and *transfer*, Δ, momenta are shared between the interacting and final partons, by means of the variables $\tilde{x}$ and α, respectively. $H_{q_0 q}$ for the constituent quark target, is related to the α-DD's, which we call $\tilde{\Phi}_{q_0 q}(\tilde{x}, \alpha, \Delta^2)$ for the constituent quark, in the following way:

$$H_{q_0 q}(x, \xi, \Delta^2) = \int_{-1}^{1} d\tilde{x} \int_{-1+|\tilde{x}|}^{1-|\tilde{x}|} \delta\left(\tilde{x} + \frac{\xi}{2}\alpha - x\right) \tilde{\Phi}_{q_0 q}(\tilde{x}, \alpha, \Delta^2) d\alpha \ . \quad (4)$$

In [13], a factorized ansatz is suggested for the DD's:

$$\tilde{\Phi}_{q_0 q}(\tilde{x}, \alpha, \Delta^2) = h_q(\tilde{x}, \alpha, \Delta^2) \Phi_{q_0 q}(\tilde{x}) F_{q_0}(\Delta^2) \quad (5)$$

where the α dependent term, $h_q(\tilde{x}, \alpha, \Delta^2)$, has the character of a mesonic amplitude, $\Phi_{q_0 q}(\tilde{x})$ represents the forward density and, eventually, $F_{q_0}(\Delta^2)$ the constituent quark f.f. In the following, we will assume for the constituent quark GPD the above factorized form, so that we need to model the three functions appearing in Eq. (5), according to the description of the reaction mechanism we have in mind. For the amplitude h_q, use will be made of one of the simple normalized forms suggested in [13]. Besides, in our approach the forward densities $\Phi_{q_0 q}(\tilde{x})$ have to be given by the standard Φ functions of the ACMP approach (see Ref. [9] for details). Eventually, as a f.f. we will take a monopole

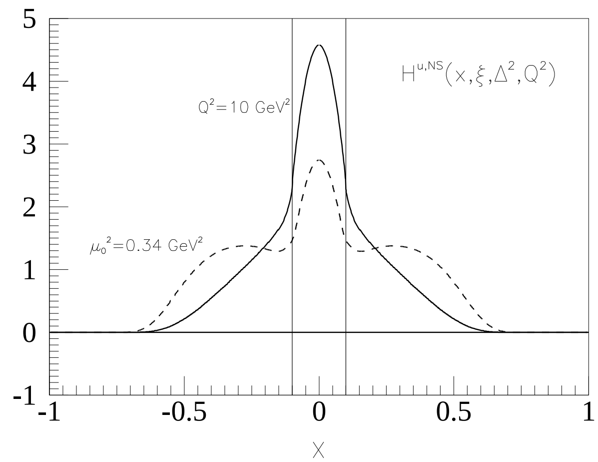

Figure 3: The NS H GPD for valence u-quarks at $\xi = 0.2$ and $\Delta^2 = -0.5$ GeV^2, at the momentum scale of the model, $\mu_0^2 = 0.34$ GeV^2 (dashed), and after NLO-QCD evolution up to $Q^2 = 10$ GeV^2 (full).

form corresponding to a constituent quark size $r_Q \simeq 0.3 fm$, supported by the analysis of [12]. By using these ingredients, Eq. (4) can be calculated and the result, when cast into Eq. (2), yields the GPD in our ACMP approach.

Results for the GPD H_q of the proton have been obtained calculating Eq. (2). $H_{q_0 q}$, has been calculated according to the model discussed above, and H_{q_0} has been evaluated according to Eq. (1), using, as an illustration, the wave functions of the Isgur and Karl (IK) model [15]. A prescription introduced in [9] has been used to correct the poor-support problem, addressed in Section 2.

Results are shown in Fig. 2. A relevant tail of the valence quarks contribution in the ERBL region is found, in agreement with other estimates [2]. The knowledge of GPDs in the ERBL region is necessary for the calculation of cross-sections. The ERBL region is accessed here, with respect to the calculation of Ref. [5], thanks to the constituent structure which has been introduced.

The results shown so far have to be related to the hadronic scale μ_0^2=0.34 GeV^2. In Fig. 3 the NLO QCD-evolution of the Non Singlet, valence u distribution, is shown. In evolving the results, the approach of Ref. [16] has been applied and a code provided by A. Freund has been used. The evolution clearly shows a strong enhancement of the ERBL region.

3 Conclusions

The aim of the talk has been to describe composite constituent quarks [11, 10] in studies of GPDs [9]. This is the continuation of an effort to construct a scheme

which describes hadrons in different kinematical and dynamical scenarios. We have developed a formalism which expresses the hadronic GPDs in terms of constituent quarks GPDs by means of appropriate convolutions. Looking at our results, we discovered that such a scheme transforms a hadronic model, in whose original description only valence quarks appear, into one containing all kinds of partons. Moreover, the starting model produces no structure in the ERBL region, while after the structure of the constituent quark has been incorporated, it does. The completeness of the x-range, for the allowed Δ^2 and ξ, of the present description, permits the calculation of cross-sections in a wide kinematical range and it can be used therefore to guide future experiments.

References

[1] D. Muller *et al.*, Fortsch. Phys. 42 (1994) 101; hep-ph/9812448.

[2] X. Ji, J. Phys. G24 (1998) 1181; A.V. Radyushkin, JLAB-THY-00-33, hep-ph/0101225; K. Goeke *et al.* Prog. Part. Nucl. Phys.47 (2001) 401; M. Diehl, hep-ph/0307382; A. Freund, hep-ph/0212017.

[3] A. Radyushkin, Phys. Lett. B 385 (1996) 333; Phys. Rev D 56 (1997) 5524.

[4] X. Ji, Phys. Rev. Lett. 78 (1997) 610; Phys. Rev. D 55 (1997) 7114.

[5] S. Scopetta and V. Vento Eur. Phys. J. A 16 (2003) 527.

[6] S. Boffi, B. Pasquini, M. Traini Nucl. Phys.B 649 (2003) 243.

[7] G. Parisi, R. Petronzio, Phys. Lett. B 62 (1976) 331; R.L. Jaffe, G.G. Ross, Phys. Lett. B 93 (1980) 313.

[8] M. Traini *et al.*, Nucl. Phys. A 614 (1997) 472.

[9] S. Scopetta and V. Vento, hep-ph/0307150.

[10] S. Scopetta *et al.* Phys. Lett. B 421 (1998) 64; Phys. Lett. B 442 (1998) 28.

[11] G. Altarelli *et al.*, Nucl. Phys. B 69 (1974) 531.

[12] R. Petronzio, S. Simula, G. Ricco, Phys. Rev. D67 (2003) 094004.

[13] A.V. Radyushkin, Phys. Rev. D 59 (1999) 014030.

[14] M. Glück, E. Reya and A. Vogt, Eur. Phys. J. C5 (1998) 461.

[15] N. Isgur and G. Karl, Phys. Rev. D 18 (1978) 4187, D 19 (1979) 2653.

[16] A.V. Belitsky, A. Freund, D. Müller Nucl. Phys. B574 (2000) 347; A. Freund, M.F. McDermott Phys. Rev. D65 (2002) 056012.